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Rigorous results for the diffusive contact processes in $d \geqslant 3$

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Abstract. The diffusive contact process is an interacting particle system on the d-dimensional hypercubic lattice. Each site can be occupied by, at most, one particle and each particle can do the following three things. (i) At rate 1 a particle will be annihilated. (ii) At rate λ a particle will give birth to a new particle at one of its $2d$ neighbour sites, if it is vacated. (iii) At rate D a particle will hop to one of its $2d$ neighbour sites, if it is vacated. For each $D \geqslant 0$, there is a critical value $\lambda_c^{(d)}(D)$ so that for $\lambda < \lambda_c^{(d)}(D)$ all particles will be annihilated with probability 1 while for $\lambda > \lambda_c^{(d)}(D)$ particles will survive with a positive probability even in the limit $t \to \infty$. In the present paper the lower and upper bounds for $\lambda_c^{(d)}(D)$ are given (theorems 1.1 and 1.2) and a lower bound for the density of particles is given in the case $\lambda > \lambda_c^{(d)}(D)$ (theorem 2.2), when the dimensionality $d \geqslant 3$. Rigorous results concluded from these theorems are shown. The crossover phenomenon for large D is discussed for the three-dimensional case.

1. Introduction

In the present paper we study the interacting particle system which we call *the diffusive contact process* (DCP). This is a combination of the basic contact process (BCP) of Harris (1974) and the exclusion process with nearest-neighbour hopping (see, for example, Liggett 1985 ch VIII). The DCP has two parameters λ and D, by which the creation rate of the contact process and the diffusion rate are determined, respectively. It is proved that the process shows two different types of long-term behaviour depending on these parameters. We will give some rigorous results on the phase diagram in the (λ, D)-plane when the spatial dimensionality $d \geqslant 3$.

At first we explain the BCP in order to introduce the problem which we will consider in this paper. The BCP ξ_t is a continuous-time Markov process defined on the d-dimensional hypercubic lattice $\mathbf{Z}^d$. At each site $x \in \mathbf{Z}^d$, a variable $\xi_t(x)$ takes values 0 and 1, representing a vacancy and a particle, respectively. Each particle can do two different things: (i) at rate 1 a particle will be annihilated; (ii) at rate $2d\lambda$ a particle will create a new particle and will choose one site at random from the $2d$ nearest-neighbour sites. If the chosen site is not occupied by another particle, then the new particle is sent to the site. However, if it is already occupied by a particle, then this creation process is suppressed. There is a critical value $\lambda_c^{(d)}$ for $d \geqslant 1$ so that for $\lambda \leqslant \lambda_c^{(d)}$ all particles will be annihilated after a sufficiently long time for any initial state (*extinction of the process*), however for $\lambda > \lambda_c^{(d)}$ particles will survive with a positive probability at any time for any non-empty initial state (*survival of the process*), where the empty state means the one with all sites vacated. As a matter of course, the critical value is determined by the balance between the creation rate and the annihilation rate. The difficulty comes from the fact that the *effective* value of the creation rate depends

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on the surrounding particle configuration. If all the nearest-neighbour sites of a site x are vacated, the rate at which a new particle is created and sent from x to some of its neighbours is $2d\lambda$ (a *bare* value). However, if some of the nearest-neighbour sites are occupied, then the rate is reduced. For example, if n nearest-neighbours are occupied, then the total creation rate is $(2d-n)\lambda$. It should be remarked that the probability to find just n particles in the nearest-neighbour sites of x depends not only on the particle configuration at x but also on the configuration at the sites which are the next-nearest-neighbours of x. In other words, the effective value of the creation rate is usually less than $2d\lambda$ and the dependence on λ may be nonlinear. This is the reason why the exact value of $\lambda_c^{(d)}$ is not yet known even for $d=1$. The reviews of the BCP are given by Liggett (1985) and Durrett (1988, 1991a).

In this paper, we introduce a diffusion process into the BCP. In the DCP each particle moves to one of its nearest-neighbour sites with rate D, if the chosen neighbour site is not occupied by another particle. The problem is to find how the critical value depends on the rate D, that is, to determine $\lambda_c^{(d)}(D)$. When D is so large, a new particle will diffuse so quickly to infinity and the creation rate will remain at the bare value. Then, in the limit $D \to \infty$, the critical value would be determined by the following simple balance: $2d\lambda = 1$, which implies $\lim_{D\to\infty} \lambda_c^{(d)}(D) = 1/(2d)$. How about the case $D < \infty$? We expect $\lambda_c^{(d)}(D)$ to be a decreasing function of D, however, it is not trivial. For example, consider the situation in which a new particle is created by a particle at x. This particle will diffuse but cannot reach infinity within a finite time period, and it will remain at some site, y. This will reduce the creation rate of particles at the neighbouring sites of y. The monotonicity of $\lambda_c^{(d)}(D)$ as a function of D has not yet been proved.

This diffusive particle system was studied by mean-field-type approximations by Katori and Konno (1992) and Matsuda *et al* (1992). Matsuda *et al* (1992) introduced the DCP as a model of population dynamics of interacting species of organisms, where the diffusion rate D means the migration rate of each individual. Jensen and Dickman (1993) studied the one-dimensional DCP in detail using the time-dependent perturbation theory and by Monte Carlo simulations. References to the field-theoretical approach to the DCP are given in Jensen and Dickman (1993).

By using the submodularity of the survival probability of the process, the present author and Konno (1992) proved the following lower bound of the critical value $\lambda_c^{(d)}(D)$ for any dimensions $d \geqslant 1$. It should be remarked that we called the DCP the single annihilation model (SAM) to emphasize the difference from the multi-particle annihilation models (Dickman 1989, 1990, Katori and Konno 1993).

Theorem 1.1 (Katori and Konno 1992). Assume that $d \geqslant 1$ and let

$$\lambda_L^{(d)}(D) = \frac{1+(2d-1)D}{(2d-1)(1+2dD)}\,. \tag{1.1}$$

Then for any $D \geqslant 0$,

$$\lambda_L^{(d)}(D) \leqslant \lambda_c^{(d)}(D)\,. \tag{1.2}$$

In the present paper we give the following upper bound when the dimensionality $d \geqslant 3$.

Theorem 1.2. Assume that $d \geqslant 3$ and let

$$\lambda_U^{(d)}(D) = \frac{1}{4d(2-G^{(d)}(0,0))}\Big[(G^{(d)}(0,0)-4dD) + \sqrt{(G^{(d)}(0,0)-4dD)^2 + 16dD(2-G^{(d)}(0,0))}\Big] \tag{1.3}$$

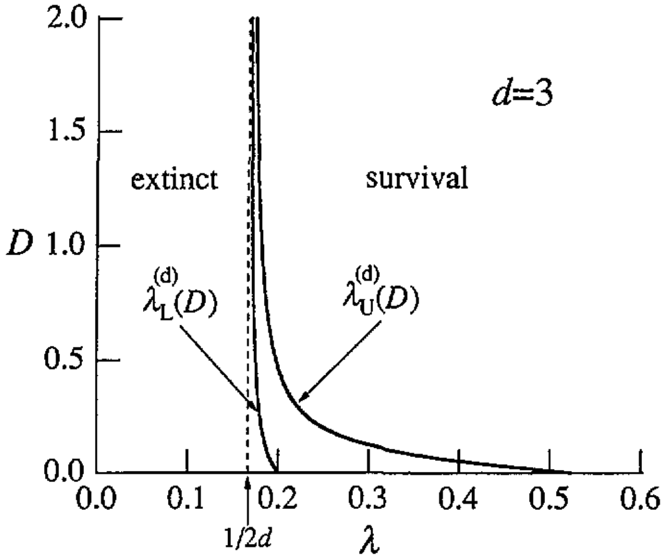

Figure 1. The numerical values of the lower and upper bounds of $\lambda_c^{(d)}(D)$ given by theorems 1.1 and 1.2 are shown for $d = 3$. The exact values of $\lambda_c^{(d)}(D)$ should be between these two curves. It should be remarked that both bounds are decreasing in D and have the same asymptote $\lambda = 1/2d$ in the limit $D \to \infty$.

where $G^{(d)}(x, y)$ is the Green function for the simple random walk on $\mathbf{Z}^d$. Then for any $D \geqslant 0$

$$\lambda_c^{(d)}(D) \leqslant \lambda_U^{(d)}(D). \tag{1.4}$$

This upper bound (1.3) is an extension for the diffusive case ($D > 0$) of the bound of Griffeath (1983) for the BCP ($D = 0$): $\lambda_c^{(d)} = \lambda_c^{(d)}(0) \leqslant G^{(d)}(0,0)/2d(2 - G^{(d)}(0,0))$ for $d \geqslant 3$. Figure 1 shows the numerical values of the bounds for $d = 3$, where $G^{(3)}(0,0) = 4\sqrt{6}/\pi^2 \times \Gamma(\frac{1}{24})\Gamma(\frac{5}{24})\Gamma(\frac{7}{24})\Gamma(\frac{11}{24}) \simeq 1.516\,386\,059\,1$ (see, for example, Itzykson and Drouffe 1989, p 21).

As shown by figure 1 both bounds are decreasing in D and have the same asymptote $\lambda = 1/2d$ in the limit $D \to \infty$. This fact proves the above-mentioned expectation, $\lim_{D\to\infty} \lambda_c^{(d)}(D) = 1/2d$. Since the value $1/2d$ is given by a simple mean-field-type approximation, we often call it the *mean-field value*. Moreover, we can conclude from the above theorems that the correction of $\lambda_c^{(d)}(D)$ to the mean-field value $1/2d$ is proportional to D^{-1} when $D \gg 1$, if $d \geqslant 3$.

Corollary 1.3. If $d \geqslant 3$, then

$$\lambda_c^{(d)}(D) - \frac{1}{2d} = c^{(d)} D^{-1} + \mathcal{O}(D^{-2}) \qquad \text{for} \quad D \gg 1 \tag{1.5}$$

where

$$\frac{1}{(2d)^2(2d-1)} \leqslant c^{(d)} \leqslant \frac{1}{(2d)^2}\big(G^{(d)}(0,0) - 1\big). \tag{1.6}$$

Recently Konno (1994, 1995) studied the asymptotic behaviour of the DCP for large D. He developed the comparison method which was used by Bramson *et al* (1989) for another modified process of the BCP. Although the proof is rather lengthy, this method is powerful for any dimensions $d \geqslant 1$. He proved that $\lambda_c^{(d)}(D) - 1/2d \simeq \text{constant} \times \varphi_d(D)$ for large D with $\varphi_1(D) = D^{-1/3}, \varphi_2(D) = |\log D|/D$ and $\varphi_d(D) = D^{-1}$ for $d \geqslant 3$. Since the local central limit theorem is used, the upper bound of $\lambda_c^{(d)}(D)$ can only be obtained for sufficiently large D by this method. On the other hand, theorem 1.2 gives the upper bound for any $D \geqslant 0$ and the asymptotic behaviour (1.5) is derived as a corollary. The numerical estimation (1.6) for the coefficient $c^{(d)}$ is a new result (for example, $0.005 < c^{(3)} < 0.015$ for $d = 3$).

By using the result of the $1/d$ expansion of $G^{(d)}(0,0)$ (see, for example, Itzykson and Drouffe 1989, p 14), we can obtain another corollary of theorems 1.1 and 1.2 for large d.

Corollary 1.4. (i) If $D = 0$, then

$$\lambda_c^{(d)}(0) = \frac{1}{2d} + \frac{c_1}{(2d)^2} + \mathcal{O}\left(\frac{1}{(2d)^3}\right) \tag{1.7}$$

with

$$1 \leqslant c_1 \leqslant 2. \tag{1.8}$$

(ii) If $D > 0$, then

$$\lambda_c^{(d)}(D) = \frac{1}{2d} + \frac{1}{D}\frac{1}{(2d)^3} + \frac{c_2}{(2d)^4} + \mathcal{O}\left(\frac{1}{(2d)^5}\right) \tag{1.9}$$

with

$$\frac{D-1}{D^2} \leqslant c_2 \leqslant \frac{6D-1}{2D^2}. \tag{1.10}$$

Since (1.9) lacks the $1/(2d)^2$ term, this corollary implies that the critical value $\lambda_c^{(d)}(D)$ approaches the mean-field value $1/2d$ as $d \to \infty$ more quickly when $D > 0$ than it does when $D = 0$.

The paper is organized as follows. In section 2 the proof of theorem 1.2 is given. There we will derive a lower bound for the density of particles $\rho_{\text{DCP}}^{(d)}(\lambda, D)$ for the survival phase, $\lambda > \lambda_c^{(d)}(D)$, in theorem 2.2. Some remarks on the critical phenomena associated with $\rho_{\text{DCP}}^{(d)}(\lambda, D)$ are given in section 3 based on theorems 1.1, 1.2 and 2.2.

2. Binary contact path process with exchange

2.1. *Griffeath's argument*

Griffeath (1983) introduced an interacting particle system called the binary contact path process (BCPP) and using it obtained the upper bound of the critical value and the lower bound of the density of particles in the survival phase for the BCP when the dimensionality $d \geqslant 3$. In order to obtain the same kind of bounds for the diffusive case, DCP, we will study the BCPP *with exchange* and will follow his argument. Here we give a brief review of Griffeath's argument.

At first we give a precise definition of the critical value $\lambda_c^{(d)}$ for the BCP. Let ν_λ be the stationary distribution of the BCP ξ_t starting from the state with all sites occupied; $\xi_0(x) = 1, {}^\forall x \in \mathbf{Z}^d$. We will represent such a state by δ_1 as an abbreviation. Consider the density of particles in this stationary distribution

$$\rho_{\text{BCP}}^{(d)}(\lambda) = \nu_\lambda\{\xi : \xi(x) = 1\} \tag{2.1}$$

which is independent of $x \in \mathbf{Z}^d$, since ν_λ is translation invariant. It is easy to prove that $\rho_{\mathrm{BCP}}^{(d)}(\lambda)$ is non-decreasing in λ and we can define $\lambda_c^{(d)}$ by

$$\lambda_c^{(d)} = \inf\left\{\lambda \geqslant 0 : \rho_{\mathrm{BCP}}^{(d)}(\lambda) > 0\right\}. \tag{2.2}$$

It is proved that any process becomes extinct with probability 1 if $\lambda \leqslant \lambda_c^{(d)}$, while if $\lambda > \lambda_c^{(d)}$ all processes starting from non-empty states have a positive probability of survival (see Liggett 1985 ch VI, Durrett 1991a). By definition (2.2), a positive lower bound for $\rho_{\mathrm{BCP}}^{(d)}(\lambda)$ gives an upper bound for $\lambda_c^{(d)}$.

The BCPP $\hat{\eta}_t$ is an interacting particle system on $\mathbf{Z}^d$ where more than one particle can exist on one site at the same time; $\hat{\eta}_t(x) \in \{0, 1, 2, \ldots\}$. Let $\{N_x(t), x \in \mathbf{Z}^d\}$ be independent rate 1 Poisson processes. At every event time t of $N_x(\cdot)$ for each site $x \in \mathbf{Z}^d$, the configuration $\hat{\eta}_{t^-}$ is replaced by the following stochastic rules. At rate $(1 + 2d\lambda)^{-1}$, $\hat{\eta}_{t^-}(x)$ is replaced by 0, and for each of $2d$ neighbours y of x, $\hat{\eta}_{t^-}(y)$ is replaced by $\hat{\eta}_{t^-}(y) + \hat{\eta}_{t^-}(x)$ at rate $\lambda(1 + 2d\lambda)^{-1}$. (Otherwise $\hat{\eta}_t(z) = \hat{\eta}_{t^-}(z)$.) If we consider the projection ζ_t given by

$$\zeta_t(x) = \begin{cases} 1 & \text{if} \quad \hat{\eta}_t(x) > 0 \\ 0 & \text{if} \quad \hat{\eta}_t(x) = 0 \end{cases} \tag{2.3}$$

then ζ_t is the BCP except for a deterministic time change. Therefore if we consider ζ_t starting from the state δ_1 and let $\rho_{\mathrm{pro}}^{(d)}$ be the density of particles in the stationary distribution of this projected process ζ_t, then we have

$$\rho_{\mathrm{pro}}^{(d)}(\lambda) = \rho_{\mathrm{BCP}}^{(d)}(\lambda). \tag{2.4}$$

We write the expectation value for the process starting from δ_1 as $E^1[\cdot]$. It is easy to find that (see appendix A for detail) for any $x \in \mathbf{Z}^d$

$$E^1\left[\hat{\eta}_t(x)\right] = \mathrm{e}^{ct} \tag{2.5}$$

with

$$c = \frac{2d\lambda - 1}{1 + 2d\lambda}. \tag{2.6}$$

Therefore if we define the process η_t by

$$\eta_t(x) = \mathrm{e}^{-ct}\hat{\eta}_t(x) \tag{2.7}$$

then

$$E^1[\eta_t(x)] = 1 \qquad {}^\forall t \geqslant 0. \tag{2.8}$$

This process η_t is called the *normalized* BCPP (NBCPP for short).

Since the NBCPP is a Markov process with (2.8), the martingale convergence theorem (see appendix B.1) can be applied and we have the following statement. If

$$E^1\left[\eta_t(x)^2\right] \leqslant {}^\exists M < \infty \qquad {}^\forall t \geqslant 0 \quad {}^\forall x \in \mathbf{Z}^d \tag{2.9}$$

then there exists a random variable $\eta_\infty \in [0, \infty)$ such that $\eta_t(x)$ converges to η_∞ almost surely:

$$\eta_t(x) \Longrightarrow \eta_\infty \qquad \text{a.s.} \qquad \text{as} \quad t \to \infty. \tag{2.10}$$

By definitions (2.7) and (2.3), if we assume that $\lambda \geqslant 1/2d$ and the initial states of the processes $\eta_t, \hat{\eta}_t$ and ζ_t are all in δ_1, then $\eta_t(x) > 0 \Rightarrow \hat{\eta}_t(x) > 0 \Rightarrow \zeta_t(x) = 1$ for each $t \geqslant 0$. Therefore, if we define

$$\rho_{\mathrm{NBCPP}}^{(d)}(\lambda) \equiv P(\eta_\infty > 0) \tag{2.11}$$

then we have by (2.4)

$$\rho^{(d)}_{\mathrm{BCP}}(\lambda) \geqslant \rho^{(d)}_{\mathrm{NBCPP}}(\lambda) \,. \tag{2.12}$$

On the other hand, the Cauchy–Schwarz inequality gives (see appendix B.2)

$$\rho^{(d)}_{\mathrm{NBCPP}}(\lambda) \geqslant \frac{(E[\eta_\infty])^2}{E[\eta_\infty^2]} \,. \tag{2.13}$$

Under conditions (2.8)–(2.10), it follows that (see appendix B.1)

$$E[\eta_\infty] = 1 \tag{2.14}$$

and

$$E\left[\eta_\infty^2\right] \leqslant \liminf_{t\to\infty} E^1\left[\eta_t(x)^2\right] \leqslant M \,. \tag{2.15}$$

Therefore if (2.9) holds for some λ, then we obtain a positive lower bound for $\rho^{(d)}_{\mathrm{BCP}}(\lambda)$,

$$\rho^{(d)}_{\mathrm{BCP}}(\lambda) \geqslant \frac{1}{M} > 0 \tag{2.16}$$

which implies $\lambda \geqslant \lambda^{(d)}_{\mathrm{c}}$.

2.2. NBCPPE η^D_t

We consider the binary contact path process with exchange (BCPPE for short) $\hat\eta^D_t$, where $\hat\eta^D_t(x) \in \{0,1,2,\ldots\}$ for each $x \in \mathbf{Z}^d$. Let $\{N_x(t), x \in \mathbf{Z}^d\}$ be independent rate 1 Poisson processes. At every event time t of $N_x(\cdot)$ for each site $x \in \mathbf{Z}^d$, $\hat\eta^D_{t-}$ is replaced by following: (i) at rate $(1+2d\lambda+2dD)^{-1}$ $\hat\eta^D_{t-}(x)$ is replaced by 0; (ii) for each of $2d$ neighbours y of x, $\hat\eta^D_{t-}(y)$ is replaced by $\hat\eta^D_{t-}(y) + \hat\eta^D_{t-}(x)$ at rate $\lambda(1+2d\lambda+2dD)^{-1}$; (iii) and for each of $2d$ neighbours y of x, $\hat\eta^D_{t-}(x)$ and $\hat\eta^D_{t-}(y)$ are exchanged by each other at rate $D(1+2d\lambda+2dD)^{-1}$. Otherwise $\hat\eta^D_t(z) = \hat\eta^D_{t-}(z)$. It is easy to see that

$$E^1\left[\hat\eta^D_t(x)\right] = \mathrm{e}^{\tilde c t} \tag{2.17}$$

with

$$\tilde c = \frac{2d\lambda - 1}{1+2d\lambda+2dD} \,. \tag{2.18}$$

Next we define the process η^D_t by

$$\eta^D_t(x) = \mathrm{e}^{-\tilde c t}\hat\eta^D_t(x) \tag{2.19}$$

and call η^D_t the *normalized* BCPPE (NBCPPE for short). Then

$$E^1\left[\eta^D_t(x)\right] = 1 \qquad {}^\forall x \in \mathbf{Z}^d \quad {}^\forall t \geqslant 0 \,. \tag{2.20}$$

As explained in appendix A, the NBCPPE η^D_t is one of the class of interacting particle systems called the *linear systems* (see Liggett 1985, ch IX). One of the reason why such systems are called the linear systems is that the time evolution of the second moments $E^\mu\left[\eta^D_t(x)\eta^D_t(0)\right]$ starting from the translational invariant initial distribution μ is determined only by the second moments. That is, we can derive the following equation for the NBCPPE η^D_t starting from the state δ_1 (see appendix A for detail):

$$\frac{\mathrm{d}}{\mathrm{d}t}E^1\left[\eta^D_t(x)\eta^D_t(0)\right] = \sum_y q(x,y)E^1\left[\eta^D_t(y)\eta^D_t(0)\right] . \tag{2.21}$$

Here $q(x,y)$ is given by the following.

When $|x-y|=1$

$$q(x,y)=\begin{cases}(2\lambda+4D)/(1+2d\lambda+2dD) & \text{if} \quad x\neq 0 \quad \text{and} \quad y\neq 0\\ 2\lambda/(1+2d\lambda+2dD) & \text{if} \quad x=0 \quad \text{or} \quad y=0\,.\end{cases} \tag{2.22}$$

When $|x-y|>1$

$$q(x,y)=\begin{cases}2D/(1+2d\lambda+2dD) & \text{if} \quad |x|=1 \quad \text{and} \quad y=-x\\ 0 & \text{otherwise}\end{cases} \tag{2.23}$$

And when $x\neq 0$

$$q(x,x)=\begin{cases}-\{4d\lambda+2(4d-1)D\}/(1+2d\lambda+2dD) & \text{if} \quad |x|=1\\ -(4d\lambda+8dD)/(1+2d\lambda+2dD) & \text{if} \quad |x|>1\end{cases} \tag{2.24}$$

and

$$q(0,0)=1-(4d\lambda+2dD)/(1+2d\lambda+2dD)\,. \tag{2.25}$$

Such linearity does not hold in usual interacting particle systems. In the corresponding equation to (2.21) of the DCP the third moments appear in the right-hand side. The fourth moments appear in the equations for the third moments and so on.

2.3. *Upper bound for the second moment*

For the linear systems whose second moments evolve following (2.21), the following useful theorem is given in the textbook of Liggett (1985, theorem 3.12 on p 445).

Theorem 2.1. Suppose that there is a strictly positive function $h(x)$ on $\mathbf{Z}^d$ such that

$$\lim_{x\to\infty} h(x)=1 \tag{2.26}$$

and

$$\sum_y q(x,y)h(y)=0 \qquad \forall x\in\mathbf{Z}^d\,. \tag{2.27}$$

Then

$$E^1\left[\eta_t^D(x)^2\right]\leqslant M \tag{2.28}$$

for any $x\in\mathbf{Z}^d$ with

$$M=\frac{h(0)}{\inf_y h(y)}<\infty\,. \tag{2.29}$$

Proof. Consider the function $f(t,x)$ for $t\in[0,\infty)$ and $x\in\mathbf{Z}^d$, which is a solution of the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}f(t,x)=\sum_y q(x,y)f(t,y) \tag{2.30}$$

with the initial condition $f(0,x)$. We define $q_t(x,y)$ such that

$$f(t,x)=\sum_y q_t(x,y)f(0,y)\,. \tag{2.31}$$

By using $q_t(x,y)$ the solution of (2.21) is given by

$$E^1\left[\eta_t^D(x)\eta_t^D(0)\right]=\sum_y q_t(x,y) \tag{2.32}$$

since $E^1\left[\eta_0^D(y)\eta_0^D(0)\right]=1$ for any $y\in\mathbf{Z}^d$. Let $x=0$, then (2.32) gives

$$E^1\left[\eta_t^D(0)^2\right]=\sum_y q_t(0,y)\,. \tag{2.33}$$

On the other hand, define $h_t(x)$ by the solution of (2.30) with the initial condition $f(0,x)=h(x)$:

$$h_t(x)=\sum_y q_t(x,y)h(y)\,. \tag{2.34}$$

Under the condition (2.27)

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}h_t(x)|_{t=0}&=\sum_y q(x,y)h_t(y)|_{t=0}\\&=\sum_y q(x,y)h(y)=0\,.\end{aligned} \tag{2.35}$$

In the same way we can show that

$$\frac{\mathrm{d}^n}{\mathrm{d}t^n}h_t(x)|_{t=0}=0 \qquad \forall n\geqslant 1 \tag{2.36}$$

which implies $h_t(x)=h(x)$ for any $t\geqslant 0$. Therefore (2.34) gives

$$\sum_y q_t(x,y)h(y)=h(x) \qquad \forall x\in\mathbf{Z}^d \quad \forall t\geqslant 0\,. \tag{2.37}$$

Let $x=0$ in this equation, we have

$$h(0)=\sum_y q_t(0,y)h(y)\geqslant\sum_y q_t(0,y)\times\inf_z h(z)\,. \tag{2.38}$$

Therefore if $h(x)$ is strictly positive,

$$\sum_y q_t(0,y)\leqslant\frac{h(0)}{\inf_z h(z)}\,. \tag{2.39}$$

The inequality (2.28) with (2.29) follows (2.33) and (2.39) for any $x\in\mathbf{Z}^d$, since $E^1\left[\eta_t^D(x)^2\right]$ is independent of $x\in\mathbf{Z}^d$ by the translation invariance of the mechanics of the process and the initial state δ_1. □

Now we try to find the function $h(x)$ which satisfies the conditions of theorem 2.1. Because $q(x,y)$ are given by (2.22)–(2.25) for the NBCPPE, (2.27) becomes

$$h(x)-\frac{1}{2d}\sum_{y:|y-x|=1}h(y)=\begin{cases}(1+2d\lambda)h(0)/4d\lambda & \text{if}\quad x=0\\ D\{h(x)-2h(0)+h(-x)\}/(2d\lambda+4dD) & \text{if}\quad |x|=1\\ 0 & \text{if}\quad |x|>1\,.\end{cases} \tag{2.40}$$

We assume that $d\geqslant 3$ and let $G^{(d)}(x,y)$ be the Green function for the simple random walk on $\mathbf{Z}^d$ which satisfies the following equations:

$$G^{(d)}(z,x)-\frac{1}{2d}\sum_{y:|y-x|=1}G^{(d)}(z,y)=\delta_{x,z} \tag{2.41}$$

$$G^{(d)}(x,y)=G^{(d)}(y,x) \tag{2.42}$$

and

$$\lim_{x\to\infty}G^{(d)}(z,x)=0 \qquad \text{for any } z \text{ with } \quad |z|<\infty \tag{2.43}$$

where $\delta_{x,y} = 1$ if $x = y$ and $\delta_{x,y} = 0$ if $x \neq y$.

By condition (2.26) we assume that the solution of (2.40) is given in the form

$$h(x) = 1 + aG^{(d)}(0,x) + b \sum_{u:|u|=1} G^{(d)}(u,x) \tag{2.44}$$

with some real a and b. Substituting (2.44) for $h(x)$ in (2.40) and using (2.41) and (2.42), we obtain

$$\begin{aligned} a &= \frac{1+2d\lambda}{4d\lambda}\left\{1 + aG^{(d)}(0,0) + b\sum_{|u|=1} G^{(d)}(0,u)\right\} \\ &= \frac{1+2D/\lambda}{4d\lambda\{1+(2d\lambda-1)D/2d\lambda^2\}/(1+2d\lambda) - G^{(d)}(0,0)} \end{aligned} \tag{2.45}$$

and

$$b = -\frac{D}{d(\lambda+2D)}a\,. \tag{2.46}$$

If we put $x = 0$ in (2.41) and use (2.42), we have

$$\sum_{|y|=1} G^{(d)}(y,z) = 2d\big(G^{(d)}(0,z) - \delta_{z,0}\big)\,. \tag{2.47}$$

Therefore (2.44) with (2.45) and (2.46) are rewritten as

$$\begin{aligned} h(x) = 1 &+ \frac{1+2D/\lambda}{4d\lambda\{1+(2d\lambda-1)D/2d\lambda^2\}/(1+2d\lambda) - G^{(d)}(0,0)} \\ &\times \frac{1}{\lambda+2D}\{\lambda G^{(d)}(0,x) + 2D\delta_{x,0}\}\,. \end{aligned} \tag{2.48}$$

If

$$\frac{4d\lambda}{1+2d\lambda}\left\{1 + \frac{2d\lambda-1}{2d\lambda^2}D\right\} - G^{(d)}(0,0) > 0 \tag{2.49}$$

then the function $h(x)$ given by (2.48) is strictly positive and satisfies all the conditions of theorem 2.1.

Since $G^{(d)}(0,x)$ is decreasing in $|x|$, $\inf_y h(y) = \lim_{y\to\infty} h(y) = 1$ by (2.43). Therefore when $d \geqslant 3$ and (2.49) is satisfied, theorem 2.1 gives us the following upper bound of the second moment for the NBCPPE.

$$E^1\big[\eta_t^D(x)^2\big] \leqslant M \tag{2.50}$$

for any $x \in \mathbf{Z}^d$ with

$$M = h(0) = \frac{4d\lambda(\lambda+2D)}{2\{2d\lambda^2 + (2d\lambda-1)D\} - \lambda(1+2d\lambda)G^{(d)}(0,0)} < \infty\,. \tag{2.51}$$

2.4. *Proof of theorem 1.2*

Let ξ_t^D be the DCP introduced in section 1. As in subsection 2.1, we define $\nu_{\lambda,D}$ by the stationary distribution of the DCP ξ_t^D starting from the state δ_1 and define

$$\rho_{\mathrm{DCP}}^{(d)}(\lambda, D) = \nu_{\lambda,D}\{\xi : \xi(x) = 1\} \tag{2.52}$$

for $\lambda \geqslant 0$, $D \geqslant 0$. As for the BCP we can define the critical value $\lambda_{\mathrm{c}}^{(d)}(D)$ for each $D \geqslant 0$ by

$$\lambda_{\mathrm{c}}^{(d)}(D) = \inf\left\{\lambda \geqslant 0 : \rho_{\mathrm{DCP}}^{(d)}(\lambda, D) > 0\right\}. \tag{2.53}$$

We can prove that for each $D \geqslant 0$ the DCP starting from any initial state becomes extinct with probability 1 if $\lambda < \lambda_c^{(d)}(D)$ but if $\lambda > \lambda_c^{(d)}(D)$ any DCP starting from non-empty initial state has a positive probability of survival forever.

Following Griffeath's argument explained in subsection 2.1, the results obtained in subsection 2.3 gives the following theorem.

Theorem 2.2. Assume that $d \geqslant 3$. If $\lambda > \lambda_U^{(d)}(D)$, where $\lambda_U^{(d)}(D)$ is given by (1.3), then the condition (2.49) is satisfied and

$$\rho_{\mathrm{DCP}}^{(d)}(\lambda, D) \geqslant \rho_{\mathrm{L}}^{(d)}(\lambda, D) > 0 \tag{2.54}$$

with

$$\rho_{\mathrm{L}}^{(d)}(\lambda, D) = \frac{1}{M} \tag{2.55}$$

where M is given by (2.51).

By the definition (2.53) this implies $\lambda_c^{(d)}(D) \leqslant \lambda_U^{(d)}(D)$.

3. Comments on the critical phenomena for large D

In this section we will give some comments on the critical phenomena of the DCP in $d \geqslant 3$ when the diffusion rate D is large. At first we consider the limit $D \to \infty$. We find that

$$\begin{aligned}\lim_{D\to\infty} \rho_{\mathrm{L}}^{(d)}(\lambda, D) &= \lim_{D\to\infty} \frac{2d\lambda - 1}{4d\lambda}\left[1 - \frac{\lambda(2d\lambda+1)(G^{(d)}(0,0)-1)}{2(2d\lambda-1)}\frac{1}{D} + \mathcal{O}\left(\frac{1}{D^2}\right)\right] \\ &= \frac{1}{2}\frac{2d\lambda - 1}{2d\lambda}.\end{aligned} \tag{3.1}$$

On the other hand, it is easy to obtain the following upper bound for $\rho_{\mathrm{DCP}}^{(d)}(\lambda, D)$ by comparing the DCP with the appropriate binary branching process:

$$\rho_{\mathrm{DCP}}^{(d)}(\lambda, D) \leqslant \frac{2d\lambda - 1}{2d\lambda} \tag{3.2}$$

for $\lambda \geqslant 1/2d$ and $D \geqslant 0$. Therefore we conclude that

$$\frac{1}{2} \leqslant \lim_{D\to\infty} \frac{\rho_{\mathrm{DCP}}^{(d)}(\lambda, D)}{(2d\lambda - 1)/2d\lambda} \leqslant 1 \tag{3.3}$$

for all $\lambda \geqslant 1/2d$.

As usual we expect that the critical exponent β is defined for the DCP as well as for the BCP by

$$\rho_{\mathrm{DCP}}^{(d)}(\lambda, D) \simeq \text{constant} \times \left(\lambda - \lambda_c^{(d)}(D)\right)^{\beta} \tag{3.4}$$

as λ approaches the critical value $\lambda_c^{(d)}(D)$ from the above. Generally speaking, β would be a function of the diffusion rate D as well as of the dimensionality d,

$$\beta = \beta(d, D). \tag{3.5}$$

The result (3.3) implies that $\lim_{D\to\infty} \rho_{\mathrm{DCP}}^{(d)}(\lambda, D) = \text{constant} \times (2d\lambda - 1)/2d\lambda$ and thus $\lim_{D\to\infty} \beta(d, D) = 1$ (the mean-field value) for $d \geqslant 3$. This statement is not new. As a matter of fact, the following statement was proved, which is stronger than (3.3) (Durrett and Neuhauser 1994, Konno 1994). If $\lambda \geqslant 1/2d$, then $\rho_{\mathrm{DCP}}^{(d)}(\lambda, D) \to (2d\lambda - 1)/2d\lambda$ as $D \to \infty$ for any $d \geqslant 1$.

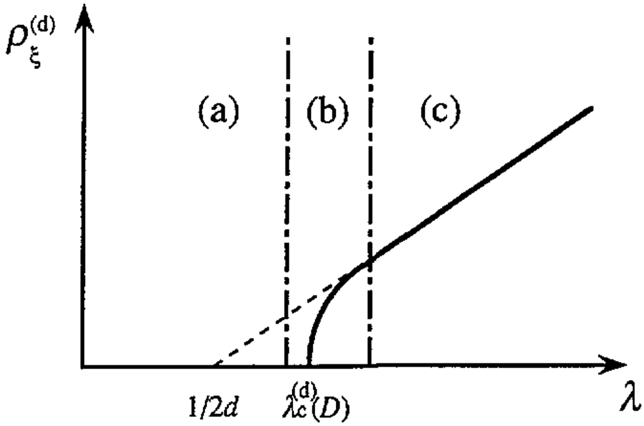

Figure 2. A schematic picture of $\rho_{\mathrm{DCP}}^{(d)}(\lambda, D)$ near the critical value $\lambda_c^{(d)}(D)$ for large D, when $d = 3$. (*a*) In the region (3.7) with small values of C, $\rho_{\mathrm{DCP}}^{(d)}(\lambda, D)$ is zero. (*b*) In the intermediate region, very near to $\lambda_c^{(d)}(D)$, the critical phenomena which belong to the same universality class as the BCP should be observed. (*c*) In the region (3.7) with large values of C, $\rho_{\mathrm{DCP}}^{(d)}(\lambda, D)$ seems to behave as constant $\times$ $(\lambda - 1/2d)$. There would be thus the crossover from the BCP-type (*b*) to the mean-field-type (*c*).

Next we consider the case when $D \gg 1$ but $D < \infty$. Theorem 2.2 and (3.1) gives that there is a constant θ $(1 < \theta < \infty)$ such that

$$\rho_{\mathrm{DCP}}^{(d)}(\lambda, D) \geqslant \frac{2d\lambda - 1}{2d\lambda}\left[1 - \left(\frac{1}{2} + \theta\frac{\lambda(2d\lambda + 1)(G^{(d)}(0,0) - 1)}{4(2d\lambda - 1)}\frac{1}{D}\right)\right] \tag{3.6}$$

for large D when $d \geqslant 3$. Here if we put

$$\lambda = \frac{1}{2d} + \frac{C}{D} \tag{3.7}$$

with some positive constant C, then we have

$$\rho_{\mathrm{DCP}}^{(d)}(\lambda, D) \geqslant \left(1 - \delta^{(d)}(C)\right)\frac{2d\lambda - 1}{2d\lambda} \tag{3.8}$$

for large D, where

$$\delta^{(d)}(C) = \frac{1}{2} + \theta'\frac{(G^{(d)}(0,0) - 1)}{2(2d)^2}\frac{1}{C} \tag{3.9}$$

with a constant θ'. Unfortunately $\delta^{(d)}(C)$ does not approach 0 as $C \to \infty$ in our estimation (3.8). However, (3.9) gives that $1 - \delta^{(d)}(C) > 0$ for sufficiently large C. Therefore, (3.8) suggests that if we observe $\rho_{\mathrm{DCP}}^{(d)}(\lambda, D)$ at the values λ given by (3.7) with sufficiently large values of C for large D, it seems to behave as constant$\times(2d\lambda - 1)/2d\lambda$: the mean-field-type behaviour. On the other hand, corollary 1.3 implies that if we put $C < \{(2d)^2(2d - 1)\}^{-1}$ in (3.7), then $\rho_{\mathrm{DCP}}^{(d)}(\lambda, D) \to 0$ as $D \to \infty$. For the intermediate values of C, the critical phenomena governed by the critical point $\lambda_c^{(d)}(D)$ would be observed.

As discussed by Jensen and Dickman (1993), it is expected that the diffusion rate D would be irrelevant for the critical phenomena: $\beta(d, D) = \beta(d, 0)$ for $D < \infty$. The upper critical dimension d_u would be 4 for the DCP as well as for the BCP. Our theorem suggests that at least when $d = 3$, we will observe the *crossover* from the critical phenomena of the BCP-type ($\beta \simeq 0.813$, Jensen 1992) to those of the mean-field-type ($\beta = 1$) for large D. Figure 2 illustrates the situation.

The crossover to the mean-field-type critical phenomena in some limit was first discussed rigorously for interacting particle systems by Bramson *et al* (1989). They studied another modified process of the BCP which can be viewed as a model of the growth of crabgrass. Recently, Konno (1994, 1995) applied their method to the DCP with large D and gave the lower bound of $\rho^{(d)}_{\mathrm{DCP}}(\lambda, D)$ in the form (3.8) for any dimensions $d \geqslant 1$ with $\delta^{(d)}(C)$ such that $\delta^{(d)}(C) \to 0$ as $C \to \infty$. His theorem suggests that the above discussed crossover will be observed in any dimensions less than d_{u}.

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Appendix A. The linear systems

The linear system with values $[0, \infty)^{\mathbf{Z}^d}$ are discussed in detail in the textbook of Liggett (1985, ch IX). Here we show some of the results given there which are used in section 2 for readers' convenience.

The linear systems γ_t on $\mathbf{Z}^d$ will be defined by using a deterministic collection of numbers $a(x, y)$ for $x, y \in \mathbf{Z}^d$ and a collection of non-negative random variables $A_x(u, v)$ with $u, v \in \mathbf{Z}^d$ for each $x \in \mathbf{Z}^d$. Let $\{N_x(t), x \in \mathbf{Z}^d\}$ be independent rate one Poisson processes. At the ith event time t of $N_x(\cdot)$, the configuration γ_{t^-} is replaced by

$$\gamma_t(u) = \sum_v A^i_x(u, v)\gamma_{t^-}(v) \tag{A.1}$$

where $\{A^i_x(u, v), u, v \in \mathbf{Z}^d\}$ are the replicas which have the same joint distribution as $\{A_x(u, v), u, v \in \mathbf{Z}^d\}$ for each $x \in \mathbf{Z}^d$ and $i = 1, 2, 3, \ldots$. Between event times of the Poisson processes, γ_t evolves according to the following linear differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\gamma_t(u) = \sum_v a(u, v)\gamma_t(v)\,. \tag{A.2}$$

The linear system γ_t defined above is a Markov process and we write the expectation of a function $f(\gamma_t)$ of the configuration γ_t starting from the initial distribution μ by $E^\mu[f(\gamma_t)]$. It is shown that the equation of motion for the expectation is given by the following:

$$\frac{\mathrm{d}}{\mathrm{d}t}E^\mu[f(\gamma_t)] = E^\mu[\Omega f(\gamma_t)] \tag{A.3}$$

where Ω is given by

$$\Omega f(\gamma) = \sum_x \{E[f(A_x\gamma)] - f(\gamma)\} + \sum_{u,v} f_u(\gamma)a(u, v)\gamma(v) \tag{A.4}$$

and called the formal generator of the process, where

$$(A_x\gamma)(u) = \sum_v A_x(u, v)\gamma(v) \tag{A.5}$$

and $f_u(\gamma) = \partial f(\gamma)/\partial\gamma(u)$. Here $E[\cdot]$ denotes the expectation with respect to the random variables $\{A_x(u, v)\}$.

Putting $f(\gamma_t) = \gamma_t(x)$ and $f(\gamma_t) = \gamma_t(x)\gamma_t(0)$ in (A.3) gives the following equations of motion for the first and the second moments.

Theorem A.1. (Theorem 1.27 on p 431 and theorem 3.1 on p 442 of Liggett 1985). Assume that the initial distribution μ is translation invariant. Then

$$\frac{\mathrm{d}}{\mathrm{d}t} E^{\mu}[\gamma_t(x)] = \sum_y s(x, y) E^{\mu}[\gamma_t(y)] \tag{A.6}$$

with

$$s(x, y) = a(x, y) + \begin{cases} E\Big[\sum_{\mathrm{u}} A_{\mathrm{u}}(x, y)\Big] & \text{if} \quad x \neq y \\ E\Big[\sum_{\mathrm{u}} \{A_{\mathrm{u}}(x, x) - 1\}\Big] & \text{if} \quad x = y \end{cases} \tag{A.7}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t} E^{\mu}[\gamma_t(x)\gamma_t(0)] = \sum_y q(x, y) E^{\mu}[\gamma_t(y)\gamma_t(0)] \tag{A.8}$$

with

$$q(x, y) = a(x, y) + a(y, x) + \sum_{u,z} E[A_z(0, u) A_z(x, u + y)] \qquad \text{for} \quad x \neq y \tag{A.9}$$

and with

$$q(x, x) = 2a(x, x) + \sum_z \Big\{ \sum_{\mathrm{u}} E[A_z(0, u) A_z(x, u + x)] - 1 \Big\}. \tag{A.10}$$

The binary contact path process with exchange (BCPPE) $\hat{\eta}_t^D$ is the linear system in which $a(x, y) = 0$ for all $x, y \in \mathbf{Z}^d$, and for each $x \in \mathbf{Z}^d$,

$$A_x(u, v) = \begin{cases} 1 & \text{if} \quad u = v \neq x \\ 0 & \text{otherwise} \end{cases} \tag{A.11}$$

with probability $(1 + 2d\lambda + 2dD)^{-1}$, and for each of the $2d$ neighbours y of x

$$A_x(u, v) = \begin{cases} 1 & \text{if} \quad u = v, \quad \text{or if} \quad u = y \quad \text{and} \quad v = x \\ 0 & \text{otherwise} \end{cases} \tag{A.12}$$

with probability $\lambda/(1 + 2d\lambda + 2dD)$ and

$$A_x(u, v) = \begin{cases} 1 & \text{if } u \neq x,\ v \neq x,\ u = v, \text{ or if } (u, v) = (x, y), \text{ or if } (u, v) = (y, x) \\ 0 & \text{otherwise} \end{cases} \tag{A.13}$$

with probability $D/(1 + 2d\lambda + 2dD)$.

The normalized BCPPE (NBCPPE) η_t^D is the one which is modified by setting

$$a(x, y) = \begin{cases} (1 - 2d\lambda)/(1 + 2d\lambda + 2dD) & \text{if} \quad x = y \\ 0 & \text{otherwise}. \end{cases} \tag{A.14}$$

Following theorem A.1 we can obtain (2.17) with (2.18), (2.20) and (2.21)–(2.25). The processes $\hat{\eta}_t$ and η_t in subsection 2.1 are the special cases for $D = 0$ of the processes $\hat{\eta}_t^D$ and η_t^D, respectively.

Appendix B. Some probability theorems

In this appendix we will list the fundamental theorems of the probability theory which we use in the subsection 2.1. For proofs and more detail, see Grimmett and Stirzaker (1992) or Durrett (1991b), for example.

B.1. Martingale convergence theorem

As explained in appendix A, the NBCPP η_t is given by (A.11)–(A.14) with $D = 0$. Since $\sum_y s(x, y) = 0$ by (A.7) for this process, we have

$$\frac{\mathrm{d}}{\mathrm{d}t} E^{\mu}[\eta_t(x)] = 0 \tag{B.1}$$

if the initial distribution μ is translation invariant. Here we define $\mathcal{F}_s$ by $\mathcal{F}_s =$ the collection of all events which happened before the time s. Then by the Markov property of the process and by (B.1), we can conclude that

$$E^{\mu}[\eta_t(x)|\mathcal{F}_s] = \eta_s(x) \qquad \text{for} \quad s < t \quad s, t \in [0, \infty) \tag{B.2}$$

if μ is translation invariant, where $E^{\mu}[\cdot|\mathcal{F}_s]$ denotes the conditional expectation given $\mathcal{F}_s$. That is, the expectation of $\eta_t(x)$ is determined to be a constant, although $\eta_t(x)$ is a random variable. Such a process is said to be a *martingale*.

For a random series $\{S_n\}$, the following convergence theorem is known.

Theorem B.1. If $\{S_n\}$ is a martingale with $E\left[S_n^2\right] < \infty$ for all n, then there exists a random variable S such that S_n converges to S almost surely.

It can be proved that such a martingale with finite second moments has an additional property called the *uniform integrability* and that the expectation also converges as

$$E[S_n] \longrightarrow E[S] \qquad \text{as} \quad n \to \infty. \tag{B.3}$$

Applying above theorems, we obtain (2.10) and (2.14) under the condition (2.9) for the NBCPP. In order to prove (2.15) we use Fatou's lemma.

Lemma B.1 (Fatou's lemma). If $\{X_n\}$ is a sequence of random variables such that $X_n \geqslant 0$, then

$$E[\liminf_{n\to\infty} X_n] \leqslant \liminf_{n\to\infty} E[X_n]. \tag{B.4}$$

Since $\liminf_{t\to\infty} \eta_t(x)^2 = \eta_\infty^2$ by (2.10), $E[\eta_\infty^2] \leqslant \lim_{t\to\infty} E^1\left[\eta_t(x)^2\right] \leqslant M$, where we have used (2.9).

B.2. Cauchy–Schwarz inequality

It is easy to prove the following inequality called the Cauchy–Schwarz inequality for random variables X and Y,

$$(E[XY])^2 \leqslant E\left[X^2\right]E\left[Y^2\right]. \tag{B.5}$$

We put $X = \eta_\infty$ and $Y = 1_{\{\eta_\infty > 0\}}$. Here 1_A denotes the indicator function of A, which gives 1 if A occurs and gives 0 otherwise. Then $E[XY] = E[\eta_\infty 1_{\{\eta_\infty > 0\}}] = E[\eta_\infty]$ since $\eta_\infty \geqslant 0$. And we observe that $E[Y^2] = E[1_{\{\eta_\infty > 0\}}] = P(\eta_\infty > 0) \equiv \rho_{\mathrm{NBCPP}}^{(d)}(\lambda)$ by definition 2.11. Therefore (B.5) gives (2.13).

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